Disaggregation of seismic drift hazard

J.W. Baker, C.A. Cornell & P. Tothong
Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA, USA

Keywords: Seismic Demand, Probabilistic Seismic Hazard, Disaggregation

ABSTRACT: The disaggregation of output from Probabilistic Seismic Hazard Analysis (PSHA) has become a frequently used tool in recent years. The output from this procedure allows one to understand the conditional probability distribution of the earthquake scenarios that contribute to seismic hazard at a specified ground motion level. In this paper, the concept of disaggregation is extended to Probabilistic Seismic Demand Analysis (PSDA)—a performance-based engineering procedure that combines ground motion hazard information with probabilistic structural response. Disaggregation of this analysis provides the distribution of ground motion intensities contributing to exceedance of a given structural response level. This information provides additional insight to the engineer, and is also useful for verifying that a sufficient range of ground motion levels has been considered the assessment of a structure. PSDA disaggregation is combined with a PSHA disaggregation to determine the distribution of Magnitude-Distance pairs (i.e., scenarios) that contribute to the exceedance of a given structural response level. A procedure is also presented for disaggregation with a vector-valued measure of ground motion intensity. The disaggregation methodology is outlined and an example analysis is performed to demonstrate the information provided.

1 INTRODUCTION

The disaggregation of output from Probabilistic Seismic Hazard Analysis (PSHA) has become a frequently used tool in recent years. The output from this procedure allows one to understand which earthquake scenarios contribute most to seismic hazard at a specified return period. This understanding is in turn valuable when selecting ground motion recordings to use in analyzing a structure.

In this paper, the concept of disaggregation is extended to Probabilistic Seismic Demand Analysis (PSDA). The PSDA procedure couples PSHA analysis with probabilistic estimates of structural response to obtain the mean annual frequency of exceeding given response levels. The result is analogous to the PSHA result, but with mean annual frequencies provided for structural response levels, rather than ground motion intensity levels. This procedure is used by the Pacific Earthquake Engineering Research (PEER) Center (Cornell and Krawinkler 2000), and forms the basis for the SAC methodology (Cornell et al. 2002), among other applications.

Disaggregation of the Probabilistic Seismic Demand Analysis provides additional information about the causal events relating to a given structural performance levels, which is useful for the same reasons that disaggregation of PSHA is useful, but is more focused on the output of direct structural interest.

The disaggregation methodology is presented, along with example results. Implications of the results are discussed.

2 DISAGGREGATION OF PSHA

The primary output from a Probabilistic Seismic Hazard Analysis is the mean annual rate of exceeding specified levels of ground motion intensity. The disaggregation of this result answers the question, “given that an earthquake ground motion with a specified level of intensity has occurred, what is the distribution of events that caused this?” Typically the parameters used to define an event are the Magnitude (*M*) and Distance (*R*) of the earthquake, and the ε value of the ground motion (a measure of the number of standard deviations by which an observed logarithmic spectral acceleration differs from the mean logarithmic spectral acceleration predicted from an attenuation relationship). If the ground motion intensity is termed an Intensity Measure, and denoted *IM*, then mathematically, this can be expressed as:

$$P\left(\left\{\begin{matrix} M=a \\ R=b \\ \varepsilon=c \end{matrix}\right\} \middle| IM=x\right) \quad (1)$$

Although the underlying *M*, *R*, and ε random variables are continuous, they are typically represented by discrete distributions in disaggregation, by dividing the range of possible values into bins, and providing the probability that *M*, *R*, and ε fall into each bin. The procedure for calculating this distribution is provided in detail elsewhere (McGuire 1995, Bazzurro and Cornell 1999).Note that the distribution of *M*, *R*, and ε can be provided conditioned on the intensity measure either equaling or exceeding *x*, although it is straightforward to convert between the two (Bazzurro 1998, p195).

Disaggregation is a standard output of many PSHA analyses, such as the U.S. Geological Survey hazard maps, (2002). Examples of this disaggregation are shown in Figures 1 and 2. The disaggregation comes from a PSHA at a site in Van Nuys, CA, near Los Angeles. The *IM* considered is spectral acceleration at a period of 0.8 seconds (denoted S_a(0.8s)). This disaggregation is a function of the site considered and the period of spectral acceleration. Additionally, the disaggregation is a function of the level of spectral acceleration considered, as illustrated by the variation between Figures 1 (for S_a(0.8s) = 0.3g) and Figure 2 (for S_a(0.8s) = 1.2g): as the ground motion intensity increases, events at nearby distances begin to dominate the disaggregation.

The disaggregation provides additional information to the seismologist and structural engineer about the distribution of events contributing to seismic hazard. This is useful, for example, for identification of representative earthquake ground motion records to use in performing dynamic analysis of a structure.

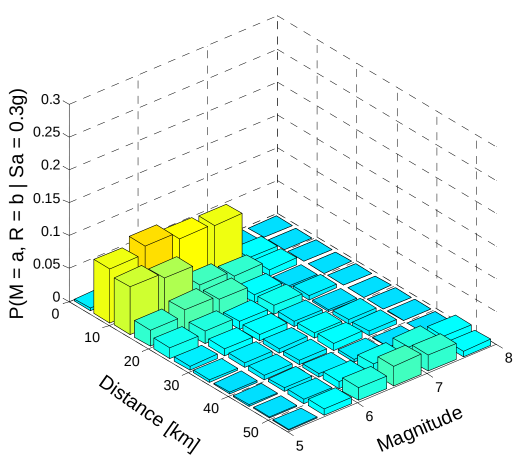

Figure 1: Distribution of (Magnitude, Distance) pairs contributing to spectral acceleration of 0.3g at a period of 0.8s.

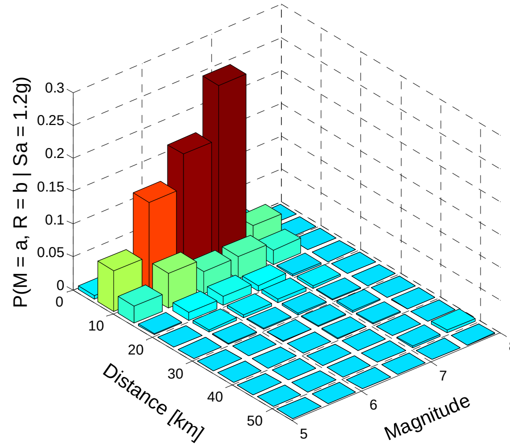

Figure 2: Distribution of (Magnitude, Distance) pairs contributing to spectral acceleration of 1.2g at a period of 0.8s.

3 PROBABILISTIC SEISMIC DEMAND ANALYSIS

The goal of Probabilistic Seismic Demand Analysis is to compute the mean annual rate of exceeding given levels of structural response. This is done by integrating probabilistic structural response over all potential levels of ground motion intensity. The information is used by, for example, the Pacific Earthquake Engineering Research Center for performance-based assessment of seismic response. In the equations below, following PEER practice, the ground motion intensity is termed an Intensity Measure, or *IM*, and the structural response is termed an Engineering Demand Parameter, or *EDP*. Using this terminology, the integral defining the mean annual frequency of exceeding a given level of *EDP* is calculated as numerically as follows:

$$\lambda_{EDP}(z) = \sum_{\text{all } x_i} P(EDP > z \mid IM = x_i) \cdot \Delta\lambda_{IM}(x_i) \quad (2)$$

where $\lambda_{IM}(x_i)$ is the mean annual frequency of exceeding a given *IM* value x_i (commonly referred to as the ground motion hazard curve) and $\Delta\lambda_{IM}(x_i) = \lambda_{IM}(x_i) - \lambda_{IM}(x_{i+1})$ is approximately the annual frequency of $IM = x_i$. The term $P(EDP>z|IM=x_i)$ represents the probability of exceeding a specified *EDP* level, *z*, given a specified level of *IM*, *x*. The term, $\lambda_{EDP}(z)$ is the mean annual frequency of exceeding a given *EDP* value *z*, sometimes referred to as the drift hazard curve. The details of this calculation are discussed elsewhere (e.g., Bazzurro et al. 1998).

4 DISAGGREGATION OF PSDA

Given the result of Equation 2, disaggregation will provide the distribution of *IM* values contributing to exceeding the given structural response level z. This is done using a Bayes' Rule calculation:

$$P(IM = x_j \mid EDP > z) = \frac{P(EDP > z \mid IM = x_j)\Delta\lambda_{IM}(x_j)}{\lambda_{EDP}(z)} \tag{3}$$

where IM has been represented by a discrete distribution rather than its true underlying continuous distribution, as was done in Equation 1 for M, R, and ε. This result is a simple extension to the calculation of Equation 2: the numerator of Equation 3 is equal to the term being summed in Equation 2. Thus, while using a computer to sum the $P(EDP>z|IM=x_i)\Delta\lambda_{IM}(x_i)$ terms, the user simply needs to copy each term to an individual element in an array. After completing the summation, the array is divided by $\lambda_{EDP}(z)$. The result is the disaggregation value of Equation 3. The ease of calculation of this information adds to its utility.

5 APPLICATION

To demonstrate the results of this procedure, and example analysis is performed. The ground motion Intensity Measure used is Spectral Acceleration at the first mode period of the structure ($S_a(T_1)$), and the Engineering Demand Parameter of interest is the maximum interstory drift ratio (the largest interstory drift ratio seen at any floor of the structure during the earthquake).

5.1 *Description of the structure*

The structure analyzed is the transverse frame of a seven-story reinforced-concrete moment-frame building located in Van Nuys, CA, which is serving as a test-bed for PEER research. The model has a first-mode period of 0.8 seconds, and contains nonlinear elements that degrade in strength and stiffness, in both shear and bending (Pincheira et al. 1999). Forty historical earthquake ground motions from California are used to analyze structural response. These records are scaled to several levels of spectral acceleration, as see in Figure 3. These results are then used to estimate the probabilistic response of the structure, at a given IM level, $P(EDP>z|IM=x_i)$.

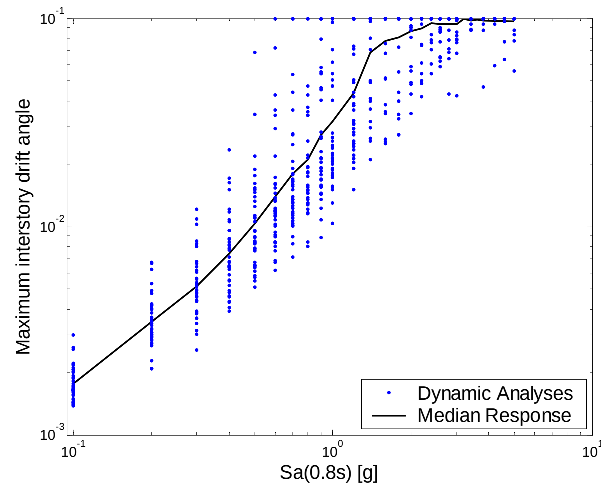

Figure 3: Spectral acceleration at 0.8s versus structural response, for a suite of earthquake ground motions scaled to several levels to spectral acceleration, and the estimated median response versus spectral acceleration..

5.2 *Ground motion hazard*

We consider the ground motion hazard at Van Nuys, CA, the site of the test structure. The ground motion hazard curve is shown in Figure 4. From this curve we can obtain $\Delta\lambda_{IM}(x_i)$, which is needed for the PSDA procedure.

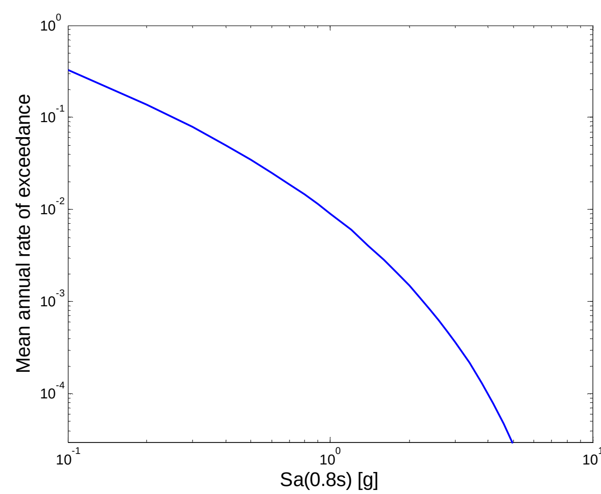

Figure 4: Probability of exceedance of spectral acceleration at 0.8s for Van Nuys, CA.

5.3 *PSDA results*

The information available from Figures 3 and 4 are combined using Equation 2 to compute the drift hazard curve $\lambda_{EDP}(z)$. The result is shown in Figure 5. From this plot we can obtain the mean annual rate of exceedance (the y-axis value) corresponding to a specified structural response level (the x-axis value). For example, the onset of significant structural damage for this structure is estimated to occur at a maximum interstory drift ratio of 0.75%, which we see to have a mean annual rate of exceedance of 0.045. Or if we are interested in a maximum interstory drift ratio of 3%, we see that the mean annual rate of exceedance is 0.009.

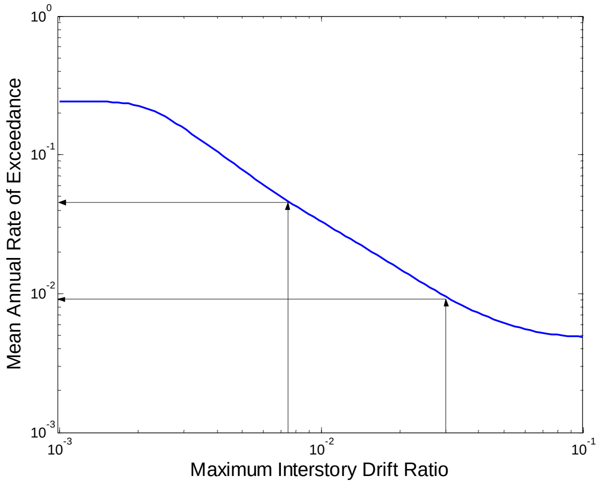

Figure 5: Mean annual rate of exceedance of maximum interstory drift ratio for the example structure at Van Nuys, CA.

5.4 *Disaggregation of PSDA results*

Disaggregation of the drift hazard curve shown in Figure 5 is performed using Equation 3. First, we consider the disaggregation for exceeding a maximum interstory drift ratio of 0.75% (corresponding to the onset of significant structural damage). The distribution of $S_a(T_1)$ values contributing to this response are shown in Figure 6. We see that a large range of $S_a(T_1)$ values contribute to this response level: large $S_a(T_1)$ values contribute because they tend to cause large responses in the structure, and smaller $S_a(T_1)$ values contribute because even though they cause lower *EDP* levels on average, they still have a probability of exceeding 0.75% and they occur much more frequently (i.e., Figure 4). Note that the $S_a(T_1)$ value with mean annual rate of exceedance of 0.045 (Figure 5) is about 0.059g (from Figure 4) which is near the mode of the distribution in Figure 6.

In Figure 7, we see the distribution of $S_a(T_1)$ values contributing to an *EDP* of 3%. These $S_a(T_1)$ values are larger than those causing an *EDP* of 0.75%, as might be expected. Disaggregation now quantifies the difference.

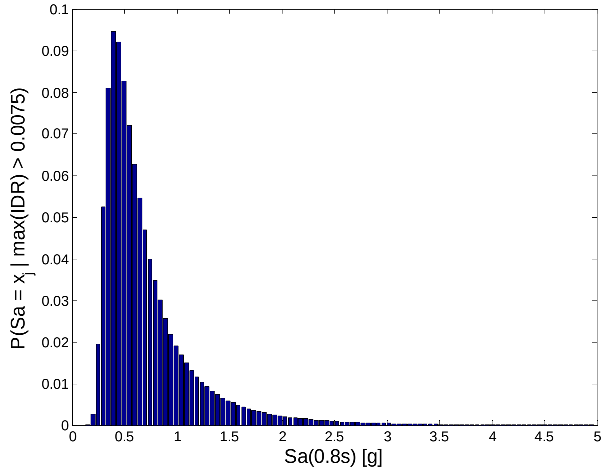

Figure 6: Distribution of $S_a(T_1)$ values contributing to exceedance of 0.75% maximum interstory drift ratio.

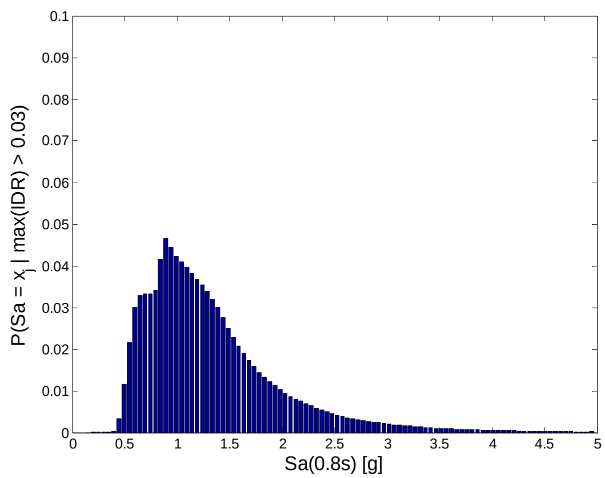

Figure 7: Distribution of $S_a(T_1)$ values contributing to exceedance of 3% maximum interstory drift ratio.

5.5 *Using disaggregation to verify the limits of integration*

This disaggregation result can also be used to verify that a sufficient range of *IM* values was integrated over in order to compute the drift hazard curve. When performing the numerical integration of Equation 2, the range of ground motion intensities (*IM*s) should be chosen to capture all of the ground motion levels that contribute to the structural responses of interest. The lower bound on *IM* should be chosen such that the small ground motions excluded do not cause significant response in the structure. The upper bound on *IM* should be chosen such that the excluded stronger records are exceedingly rare, and thus do not contribute to the response of the structure.

In the example presented here, the range of $S_a(T_1)$ values was 0.1g to 5.0g. But if a range of 0.1g to 2g had been used instead, the result would be an underestimation of the mean annual rate of exceeding some levels of structural response. This can be clearly seen when performing disaggregation of the drift hazard. In Figure 8, we see the distribution of $S_a(T_1)$ values given *EDP*>3%, when this reduced range of integration is used. The right tail of the distribution is missing because $S_a(T_1)$ values larger than 2g were not included in the range of integration. This is very clear when examining Figure 8, but would not be obvious without this disaggregation result. After identifying the problem in Figure 8, an engineer could then repeat the drift hazard calculation with an appropriate range of *IM* values, and verify that the disaggregation looks like Figure 7, rather than Figure 8.

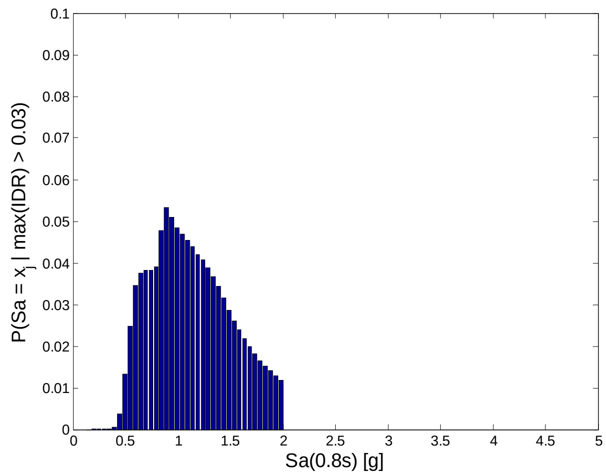

Figure 8: Distribution of $S_a(T_1)$ values contributing to exceedance of 0.75% maximum interstory drift. Only $S_a(T_1)$ values between 0.1g to 2g were used to compute the PSDA, illustrating the effect of using too small of a range of *IM*s.

6 NESTED DISAGGREGATIONS

The disaggregation of PSDA results provides the distribution of $S_a(T_1)$ values given *EDP*>x. For each of these $S_a(T_1)$ values, it is possible to use the PSHA disaggregation to find the conditional distribution of *M*, *R*, and ε. By nesting these disaggregations, it is possible to obtain the earthquake events contributing to exceedance of a specified *EDP* level.

The following equation depends upon $S_a(T_1)$ being *sufficient* for prediction of *EDP* with respect to *M*, *R*, and ε (i.e., given knowledge of $S_a(T_1)$, knowledge of *M*, *R*, and ε does not provide further information about the structural response, *EDP*). The idea of sufficiency is described by Luco and Cornell (2004) who verify approximate sufficiency of $S_a(T_1)$ with respect to *M* and *R*. However, $S_a(T_1)$ is not sufficient with respect to ε (Baker and Cornell 2004b), and so this simple nested disaggregation procedure is only valid for *M* and *R*. In the case of a variable such as ε that is not sufficient, a nested disaggregation is possible, but the computation is more difficult.

Under the sufficiency condition, the distribution of *M* and *R*, given *EDP*>*z* can be calculated using this simple form of the Total Probability Theorem:

$$P\left(\begin{Bmatrix} M=a \\ R=b \end{Bmatrix} \middle| EDP>z\right) = \sum_{\text{all } x_i} P\left(\begin{Bmatrix} M=a \\ R=b \end{Bmatrix} \middle| IM=x_i\right) \cdot P(IM=x_i \mid EDP>z) \quad (4)$$

The first factor in the summation comes from Equation 1 (after simply removing ε from the disaggregation). The second factor comes from Equation 3. The disaggregation of *M* and *R* for the example problem is shown in Figure 9, conditioned on *EDP*>0.75%.

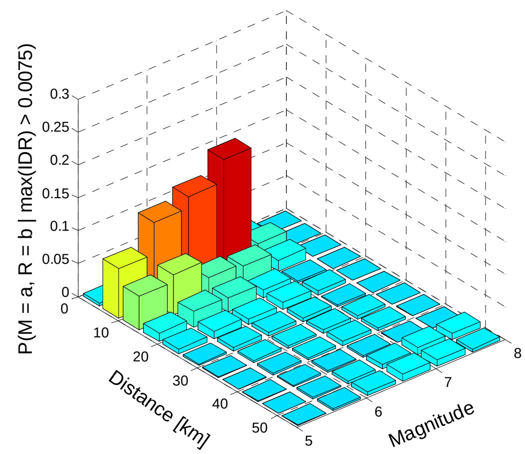

Figure 9: Distribution of (Magnitude, Distance) pairs contributing to exceedance of 0.75% maximum interstory drift.

It is illustrative to compare the disaggregation of Figure 9 to earlier disaggregation results. From Figure 6, we see that the event *EDP*>0.75% is caused primarily by $S_a(T_1)$ values between 0.3g and 1.2g. The PSHA disaggregation at these levels of $S_a(T_1)$ are given in Figures 1 and 2. The disaggregation of Equation 4 has incorporated an average of all of the disaggregations of $S_a(T_1)$, weighted by the contribution of $S_a(T_1)$ to the probability of exceedance of the *EDP* level. Thus, it is reasonable that the PSDA disaggregation of Figure 9 looks similar to an average of the PSHA disaggregations of Figures 1 and 2. These conclusions are of course site specific.

7 GENERALIZATION OF PSDA DISAGGREGATION TO A VECTOR-VALUED IM

The disaggregation procedure of Equation 3 only considers the case where the Intensity Measure consists of a single parameter. But recent work has shown the benefit of "vector-valued" Intensity Measures consisting of more than one parameter (Baker and Cornell 2004a, b). For this case, the disaggregation procedure is easily generalized:

$$P\left(\begin{Bmatrix} IM_1=x_j \\ IM_2=y_k \end{Bmatrix} \mid EDP>z\right) = \frac{P(EDP>z \mid IM=x_j, IM_2=y_k)\Delta\lambda_{IM}(x_j,y_k)}{\lambda_{EDP}(z)} \quad (5)$$

where $P(EDP>z|\ IM_1=x_i,\ IM_2=y_k)$ is a vector-based probabilistic *EDP* prediction, and $\Delta\lambda_{IM}(x_i, y_k)$ is a vector-valued hazard analysis result. The term $\lambda_{EDP}(z)$ is the drift hazard curve calculated using the vector-valued *IM* procedure. The calculation of these terms is discussed in detail by Baker and Cornell (2004a). The disaggregation is generalized to Intensity Measures consisting of more than two parameters in the same way.

Now the disaggregation output will be a matrix of probabilities corresponding to the probability that a set of *IM* parameters is that which caused the given *EDP* level to be exceeded.

An example result is shown in Figure 10. This is the disaggregated drift hazard for the same structure as used in the previous examples, but using a vector-valued *IM* consisting of $S_a(T_1)$ and ε as proposed by Baker and Cornell (2004b). This is the same ε value that arose earlier in discussions of PSHA disaggregation. It should be noted that the $\Delta S_a(T_1)$ and $\Delta\varepsilon$ intervals in Figure 10 are quite small. This was done to clearly illustrate the shape of the density function, but in general it is not necessary to have such fine interval spacing.

It should be noted that the disaggregation is dominated by positive ε values, especially when paired with large $S_a(T_1)$ values. This is because the ground motion hazard is increasingly dominated by positive ε values as $S_a(T_1)$ increases. Large $S_a(T_1)$ values and negative ε values cannot occur simultaneously, and thus they cannot contribute to the drift hazard. Baker and Cornell (2004b) have shown, however, that large positive ε values tend to be associated with comparatively benign non-linear response amplitudes (for the same $S_a(T_1)$ value).

The drift hazard disaggregation provides special insight into the effect of including ε in an intensity measure. The disaggregation procedure may also provide insight regarding the use of future candidates for improved intensity measures.

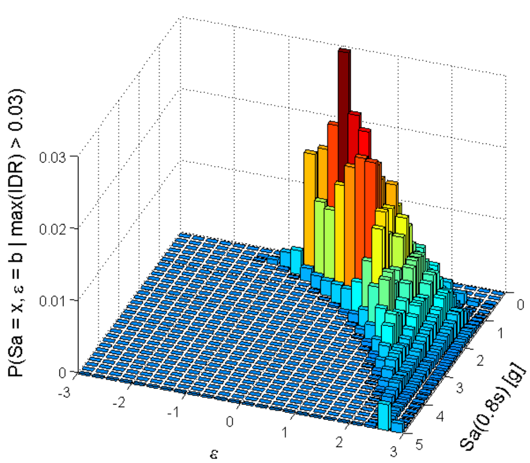

Figure 10: Distribution of ($S_a(T_1)$, ε) pairs contributing to exceedance of 3% maximum interstory drift.

8 CONCLUSIONS

The principle of disaggregation of Probabilistic Seismic Hazard Analysis has been extended to disaggregation of Probabilistic Seismic Demand Analysis. The calculation is made using Bayes' Rule, and is seen to be quite simple.

Despite its simplicity, the information produced is useful to the engineer. The range of ground motion intensities contributing to a given level of structural response is available in a simple visual display. Further disaggregation allows the engineer to understand the earthquake sources which contribute to exceedance of a given response—a result which is likely not intuitive otherwise. The disaggregation also allows the engineer to visually verify that a sufficient range of ground motion intensity levels were included in the analysis, which is a check not easily available otherwise.

Because of the simplicity of calculation, and the potential benefit of this knowledge, engineers are encouraged to calculate the disaggregation when performing Probabilistic Seismic Demand Analysis. This will provide increased understanding of the earthquake motions and events contributing to response, without requiring significantly more effort.

ACKNOWLEDGEMENTS

This work was supported primarily by the Earthquake Engineering Research Centers Program of the National Science Foundation, under Award Number EEC-9701568 through the Pacific Earthquake Engineering Research Center (PEER). Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation.

REFERENCES

"U.S. Geological Survey Earthquake Hazard Maps". 2002. http://eqhazmaps.usgs.gov/ (accessed November 10, 2004).

Baker J.W., Cornell C.A. 2004a. Choice of a Vector of Ground Motion Intensity Measures for Seismic Demand Hazard Analysis. in *Proceedings, 13th World Conference on Earthquake Engineering, Vancouver, Canada*, pp. 15p.

Baker J.W., Cornell C.A. 2004b. A Vector-Valued Ground Motion Intensity Measure Consisting of Spectral Acceleration and Epsilon. *Earthquake Engineering & Structural Dynamics (Submitted)*.

Bazzurro P. 1998. *Probabilistic Seismic Demand Analysis*. Stanford University. Stanford, CA. 329 pp.

Bazzurro P., Cornell C.A. 1999. Disaggregation of Seismic Hazard. *Bulletin of the Seismological Society of America*. Vol. 89 (No. 2): 501-520.

Bazzurro P., Cornell C.A., Shome N., Carballo J.E. 1998. Three Proposals for Characterizing MDOF Nonlinear

Seismic Response. *Journal of Structural Engineering*. Vol. 124 (No. 11): 9.

Cornell C.A., Jalayer F., Hamburger R.O., Foutch D.A. 2002. Probabilistic Basis for 2000 SAC Federal Emergency Management Agency Steel Moment Frame Guidelines. *Journal of Structural Engineering*. Vol. 128 (No. 4): 526-533.

Cornell C.A., Krawinkler H. 2000. Progress and Challenges in Seismic Performance Assessment. *PEER Center News*. Vol. 3 (No. 2). http://peer.berkeley.edu/news/2000spring/performance.html (accessed November 10, 2004).

Luco N., Cornell C.A. 2004. Structure-Specific Scalar Intensity Measures for near-Source and Ordinary Earthquake Ground Motions. *Earthquake Spectra (Under revision for publication)*.

McGuire R.K. 1995. Probabilistic Seismic Hazard Analysis and Design Earthquakes: Closing the Loop. *Bulletin of the Seismological Society of America*. Vol. 85 (No. 5): 1275-1284.

Pincheira J.A., Dotiwala F.S., D'Souza J.T. 1999. Seismic Analysis of Older Reinforced Concrete Columns. *Earthquake Spectra*. Vol. 15 (No. 2): 245-272.